	MIDTERM EXAM 2	
Name, Surname:	Department:	GRADE
Student No:	Course: Calculus I	
Signature:	Exam Date: 24/12/2019	

Solve 4 of the 6 questions. Each problem is 25 points. Duration is 55 minutes.

1. A cube's surface area increases at the rate of 72 cm^2/sec. At what rate is the cube's volume changing when the edge length is $x = 3cm$? $\boxed{}$

Solution: $x = x(t)$ $A = 6x^2$, $V = x^3$.

$$72 = \frac{dA}{dt} = 12x\frac{dx}{dt} \implies \frac{dx}{dt} = 2 \text{ when } x = 3.$$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt} = 3\cdot 3^2\cdot 2 = 54$$

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2. $\lim_{x\to 0} \frac{x-\sin x}{x-\tan x} =$ $\boxed{}$

Solution: By L'Hopital's Rule

$$\lim_{x\to 0}\frac{x-\sin x}{x-\tan x} = \lim_{x\to 0}\frac{1-\cos x}{1-(1+\tan^2 x)} = \lim_{x\to 0}\frac{\sin x}{-2\tan x(1+\tan^2 x)} = \lim_{x\to 0}\frac{1}{-2\frac{1}{\cos x}(1+\tan^2 x)} = \frac{-1}{2}$$

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3. $\int x \arctan x dx =$ $\boxed{}$

Solution: Integrating by parts:

$$u = \arctan x, \qquad dv = xdx \implies du = \frac{dx}{1+x^2}, \qquad v = \frac{x^2}{2}$$

$$\int x\arctan x dx = \arctan x\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\frac{dx}{1+x^2} = \arctan x\cdot\frac{x^2}{2} - \frac{1}{2}\int\frac{(1+x^2-1)}{1+x^2}dx$$
$$= \arctan x\cdot\frac{x^2}{2} - \frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx = \arctan x\cdot\frac{x^2}{2} - \frac{1}{2}(x-\arctan x) + C$$

4. Use the linearization of a suitable function to approximate $(1.03)^{40}$. $\boxed{}$

Solution: $f(x) = x^{40}$, $f(1) = 1$, $f'(x) = 40x^{39}$, $f'(1) = 40$. The linearization is

$$L(x) = f(1) + f'(1)(x-1) = 1 + 40(x-1) \implies f(1.03) \approx L(1.03) = 1 + 40\cdot 0.03 = 1 + 1.2 = 2.2$$

5. Identify the coordinates of any local and absolute extreme points and inflection points of $y = xe^{-x}$. Graph the function.

Solution:

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$$

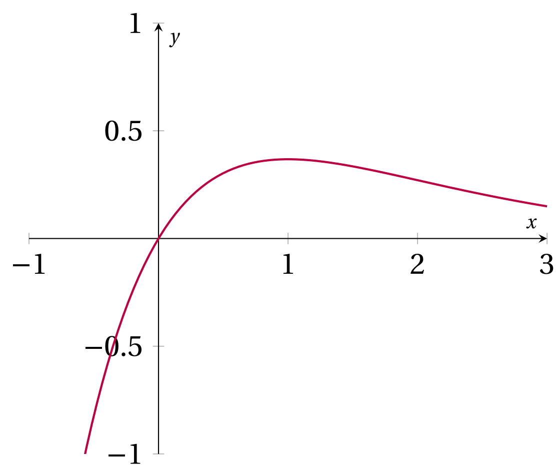
$$\lim_{x\to-\infty} y = -\infty$$

$$y' = e^{-x}(1-x)$$

Only critical point is $x = 1$. $y' > 0$ and y is increasing if $x < 1$ and $y' < 0$ and y is decreasing if $x > 1$. So $x = 1$ is an **absolute maximum**.

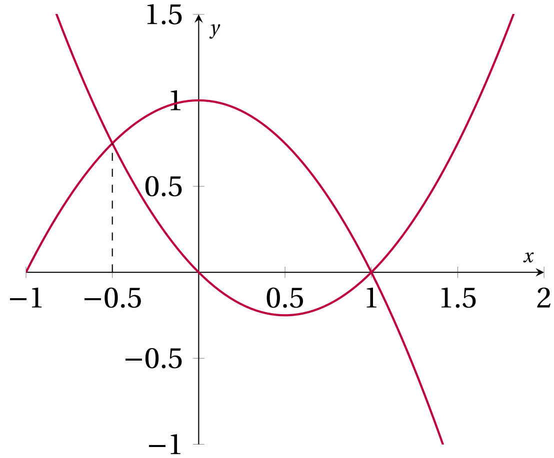
$$y'' = e^{-x}(x-2)$$

$y'' < 0$ and y is concave down if $x < 2$ and $y'' > 0$ and y is concave up if $x > 2$. $(2, 2/e^2)$ is an **inflection point.**



6. (A) Sketch the region bounded by $y = x^2 - x$ and $y = 1 - x^2$. (B) Find its area.

Solution:



The graph intersection is at $x^2 - x = 1 - x^2$, that is $2x^2 - x - 1 = 0$ which gives $x = 1$, $x = -\frac{1}{2}$.

The area between the graphs is

$$\int_{-1/2}^{1} \big((1-x^2) - (x^2 - x)\big)\, dx = \int_{-1/2}^{1} \big(1 + x - 2x^2\big)\, dx = x + \frac{x^2}{2} - 2\frac{x^3}{3}\bigg|_{x=-1/2}^{1}$$

$$= 1 + \frac{1}{2} - \frac{2}{3} - \left(-\frac{1}{2} + \frac{1}{8} + \frac{1}{12}\right) = \frac{9}{8}$$